

V.

Analysis Geometrica, sive nova & vera Methodus Resolvendi, tam Problemata Geometrica, quam Arithmeticas Quaestiones. Pars prima, de Planis; Authore D. Antonio Hugone de Omerique Sanlucarense. Sold by Sam. Smith and Benj. Walford at the Prince's in St. Paul's Church-yard London.

THE Author of this Book being of opinion that the Method of deducing Geometric Demonstrations from an Algebraic Calculation, is forc'd and unnatural, has studied how to find an Analysis purely Geometrical, from which a Synthesis might easily be deriv'd, according to the Method of the Antients.

He begins with an Introduction consisting of about twenty Geometric Propositions; which are so many Lemmas, in order to make his Analysis the more easy; the chief Proposition of his Introduction; and which he has occasion to use most, is this: *To find two lines whose sum or difference is given, that shall be reciprocal to two given lines*; this comprehending the Construction of Quadratic Equations. He divides the rest of his Book into Four Parts. In the First he considers those Problems that are solv'd by simple Proportions. In the 2^d. he considers those that are solv'd by using Compound Ratio. In the 3^d. he resolves those wherein it is necessary to consider Quantities connected by the Signs + and —, And in the 4th. he considers Indeterminate Problems.

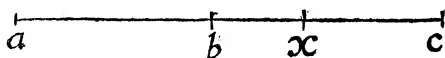
He Prefixes to his First Part some General Rules how to proceed in a Geometric Investigation; and because these Rules contain what is most material in his Method.

we think it not improper to relate 'em as he has laid 'em down himself.

10. An unknown Line is always terminated in an unknown Point; hence to avoid confusion, the unknown Points ought to be Denoted with the last Letters of the Alphabet *v, z, y, x, &c.* to distinguish 'em from the known Points *a, b, c, d, &c.* and if there is occasion, one and the same Point may be denoted with two Letters, when a known and unknown Line concur in it.

First Definition.

Additive Ratio is that whose Terms are dispos'd to Addition, that is, to Composition. *Subtractive Ratio* is that whose Terms are dispos'd to Subtraction, that is, to Division.

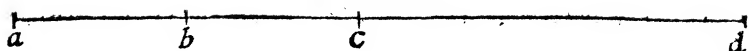


Let the Line *ac*, be divided in the Points *b*, and *x*, the Ratio between *ab*, and *bx*, is *Additive*; because the Terms *ab*, and *bx*, compose the whole *ax*; but the Ratio between *ax* and *bx* is *Subtractive*, because the Terms *ax*, and *bx*, differ by the Line *ab*.

20. The same order of the Letters which is in the Figure, ought to be kept in your Analysis, that so by meer Inspection you may know whether the Ratio is *Additive* or *Subtractive*; and consequently whether you ought to Compose or Divide.

30. When you are to argue by Proportions, and the Proportion lies in a Right Line, you have no other way to proceed on but by Composition or Division: Therefore if both Ratios are Additive, you must argue by Composition; if both Subtractive, by Division; so as always to use that way of arguing which is the fittest for the preservation of those Terms that are known; but when one Ratio is Additive and th'other Subtractive, the Additive must either be made Subtractive, or the Subtractive Additive; Now this change it wrought by repeating either Term.

For



For if we design to change the Additive Ratio of ab to bd , into Subtractive, let bc be made equal to ab , and thus the Ratio of bc to bd , that is, of ab to bd , will be Subtractive; and likewise, if the Subtractive Ratio of bd to bc was to be made Additive, it is but making ab equal to bc .

4°. This is always to be observed, when the Terms of the Ratio which is to be reduc'd, are known; but if they are unknown, and their Sum or Difference is known, it is often convenient to use the 7th. and 8th. Proposition of the Introduction by means of which the difference of the Terms of an Additive Ratio, or the sum of the Terms of a Subtractive one, may be express'd, whence you may argue by Division or Composition. Now the 7th. Proposition of the Introduction is this; If a Right Line is Divided into two equal Parts, and into two unequal Parts, the middle part is the half difference of the unequal parts. The 8th. Proposition is this; If a Right Line is Divided into two equal parts, and a Right Line is added to it, that which is compounded of the half and of the Line added, is the half sum of the Line that is added, and of that which is compounded of the whole and the Line added.

Second Definition.

That Ratio we call Common which is Common to two Proportions whether it be Direct or Reciprocal; Let there be two Proportions $a : b :: d, e$, and $b, c :: e, l$, having the same Terms b and e , and constituting a Direct Ratio, this Ratio we call Common, because it is Common to both Proportions: In like manner let there be two Proportions $a, b :: e, l$ and $b, c :: d, e$, each having the same Terms b and e which constitute a Reciprocal Ratio, this Ratio we call Common, because it is Common to both Proportions.

50. Therefore if two Proportions have a Common Ratio, we may argue by Equality; but if a Common Ratio is wanting, it must be introduc'd, that we may proceed farther, which will be done by the Reduction of some Ratio into another equal to it.

Likewise if a Proportion lies in a Triangle or any other Figure, you must use a new Proportion by repeating some Angle, that is, by changing its Position, that so you may have two equal Terms in two different Proportions, and so may argue by Equality: Hence it is evident that, that Angle ought to be transposed, which together with the other Angles and Sides of the Figure, shews the most convenient similitude of Triangles.

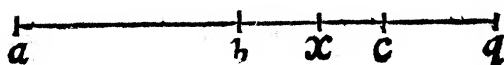
60. Now what is sought being assum'd as granted, all our endeavours must be to retain in arguing those magnitudes which are already known, and to extinguish as much as we can the unknown Point, and the Analyst understanding where to use Additive or Subtractive Ratio in one Proportion, and how to Introduce a Common Ratio in two Proportions, if it be wanting, will come to the end of this Resolution by necessary consequences: Now this end is obtain'd when the unknown Magnitude is found equal to some known Magnitude, or the unknown Point is in one Term, which is a 4th. Proportional, or in two Terms either Means or Extreams whose sum or difference is known, for a 4th. Proportional, or two Reciprocals will do it.

70. The Analysis being ended, the order of the Construction and Demonstration is evident, for nothing else is required for the Construction, but what has, or is suppos'd to have been done in the Analysis, and for the Demonstration, nothing but to begin from the end of the Analysis and proceed to the beginning of it, observing that where the Analysis argues by Alternate or Inverted Propositions, the Synthesis argues by the same,
and

and that where the Analysis Compounds, the Synthesis Divides, and *vice versa*.

But to make those Rules more useful, it won't be amiss to shew the applications he has made of 'em in the solution of some Problems, and because there is a great variety of 'em in his Book, we will chuse a few of the most remarkable as Rules in cases of the like nature.

P R O B L E M.



The Line *ac* being divided at pleasure in *b* to divide it again in *x* between *b* and *c* so that *ax* *xc*, *bx* be proportional.

Analysis.

Let therefore	$ax, xc :: xc, bx.$
and <i>Componendo</i>	$ac, xc :: bc, bx.$
and <i>Alternando</i>	$ac, bc :: xc, bx.$
Let <i>cq</i> be made = <i>bc</i>	, cq
and <i>Componendo</i>	$aq, cq :: bc, bx.$

Therefore the Problem is solv'd.

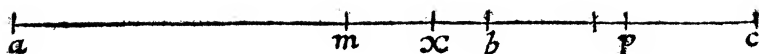
Construction.

Let the Construction be made as before.

Demonstration.

For since, by the Construction, *aq* is to *cq* as *bc* to *bx*. Therefore *Dividendo* *ac* is to *cq* that is to *bc*, as *xc* to *bx* and *Alternando* *ac* is to *xc*, as *bc* to *bx*. Therefore *Dividendo* *ax*, is to *xc* as *xc* to *bx*, which was to be done.

PROBLEM



P R O B L E M

The Line ac being Divided in b to Divide it again in x between a and b so that ax, xc, xb be Proportional. Now because in the Proportion $ax, xc :: ax, xb$, the first Ratio is *Additive* and the second *Subtractive* it is evident that the *Additive* must either be made *Subtractive*, or the *Subtractive Additive*. But because the Terms are unknown, let ac be bisected in m , and $2mx$ will be the Difference of the Parts ax, xc ; likewise let bc be bisected in p , and $2xp$ will be the sum of the Parts xc and xb ; whence one may proceed by Composition or Division.

Analysis.

Let	$ax, xc :: xc, xb$
Therof. <i>Componendo</i>	$ac, xc :: 2xp, xb$
and <i>half. the Antecedents</i>	$mc, xc :: xp, xb$
and <i>Convertendo</i>	$mc, mx :: xp, bp$

Therefore the Problem is solv'd. Because the Point x being only in the middle Terms, we can proceed no farther. And because there is nothing from whence we may infer which of the two mx and xp is the greatest, it will be in our choice to take mx either for the greatest or the least part, and there will be two Solutions for which there is one Demonstration.

Construction and Demonstration.

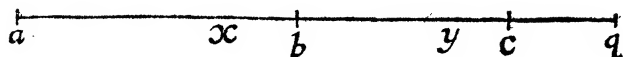
Let ac be bisected in m and bc in p , and to mc and bp or pc let two Reciprocals mx and xp be found whose sum be mp , I say the thing is done.

For by the Construction $mc, mx :: xp, bp$, Therefore *Convertendo* $mc, xc :: xp, xb$ and doubling the Antecedents $ac, xc :: 2xp, xb$, but $2xp$ is the sum of

of

of the Terms xc and xb ; therefore *Dividendo* ac , $xc::$
 xe , xb , which was to be done.

P R O B L E M.



To Divide the given Lines ab bc in x and y so
 that ay be to xc as f to g and xb to yc as h to k .

Conditions.

$$\begin{array}{l} ay \quad xc :: f, g \\ \text{and } xb \quad yc :: h, k. \end{array}$$

Analysis.

Let therefore $ay, xc :: f, g$.
 and also $xb \quad yc :: h, k$.
 or bc, cq .

And as the sum of the Antecedents to the sum of the
 Consequents, so one Antecedent to its Consequent.

Therefore $xc, yq :: h, k$.
 or

Therefore by Equality $ay, yq :: f, l$.

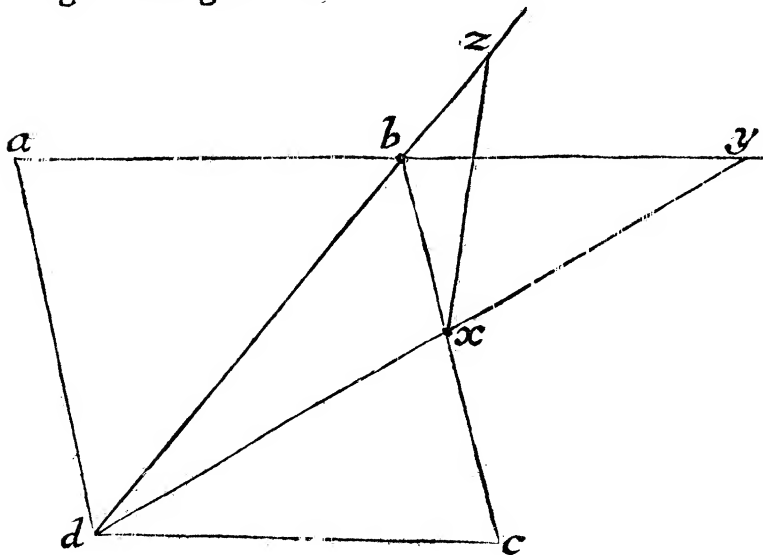
Construction and Demonstration.

Let h be to k , as bc to cq , and so g to l . Let aq be
 be Divided in y in the Ratio of f to l , and let ay be
 to xc as f to g . I say that $xb, yc :: h, k$. for since
 by the Construction $ay \quad yq :: f, l$; and ay to xc as f
 to g : by Equality xc will be to yq , as g to l that is
 as bc to cq and because the difference of the Antecedents
 is to the difference of the Consequents, as one Antecedent
 to its Consequent, xb will be to yc as bc to cq , that is, as
 h to k , which was to be done.

P R O B L E M.

A Square or Rhombus $a b c d$ being given to
 draw

draw from the Angle d to the opposite side produc'd ab a right line dxy , and to make xy equal to a right Line given m .



Let therefore xy be equal to m .
 by the 2^d. of the 6th. Book of Euclid $ab, dy :: dx, xy$.
 Let the Angle dxx be $= dby$.
 and because the Triangles dxx, dby are Similar,
 $db, by :: dx, xz$.
 Therefore by Equality $db, ab :: xy, xz$.
 But the Angle $xbz = dby$ or dxx .
 Therefore the Triangles dxx, xbx are Similar
 Therefore $dz, xz :: xz, bz$.

Construction and Demonstration.

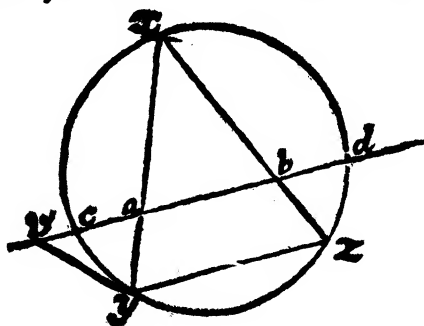
Let db be to ab , as m to g , and let dz, bz whose difference is db be found reciprocal to g . Set off from the point z the Line zx equal to g , and through x draw dxy , I say that xy is equal to the given line m .

For since by the Construction dz is to g as g to bz , that is dz is to xz as xz to bz :, The Triangles dzx, bzx will

will be Similar, Therefore the Angle dxz will be equal to the Angle xbz , that is, to the Angle dbz (for the Angles dbz and xbz are equal, because dbc in a Square or Rhombus is equal to the Angle abd , or its equal ybz , hence adding the common Angle xbz , the Angles dbz xbz will be equal.) Therefore since the Triangles dxz , dbz have the Angles dxz and dbz equal, and the Angle bdx common, they will be similar, and therefore db will be to bz as dx to xz that is to g ; but because ad , bx are parallel, ab will be to bz as dx to xy . Therefore by Equality ab is to db as g to xy . But by the Construction ab is to db as g to m , Therefore xy is equal to m . Which was to be done.

P R O B L E M.

A Circle xyz being given by Position, and two Points in it a and b being given, to draw the Lines ax , xb so that yz shall be Parallel to ab .



A N A L Y S I S.

Let therefore	yz be parallel to ab
Therefore the Angle	$abx = yzx$
Let the Angle	ayv be made $= abx$
Therefore the Angle	$ayv = yzx$
Therefore	x, v, y, b , are in a Circle
Therefore the Rectangle	$vay = xay$
But the Rectangle	$xay =$ any Rectangle through a
Theref. the Rectangle	$vab =$ any Rectangle through a .

Construction and Demonstration.

Let the Rectangle vab be made equal to any Rectangle through a such as cad , let the Tangent vy be drawn
 K k k through

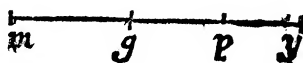
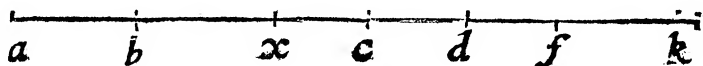
through a let the line yx , and through b the line xz be drawn, let yz be join'd, I say that yz is parallel to ab .

For since the Rectangle vab has been made equal to cad , and xay is equal to the same, the Rectangles vab xay will be equal: Therefore the points x, v, y, b , will be in a Circle, and the Angles ayv, abx upon the same Line xv will be equal, but because vy touches the Circle xyz and xy cuts it, the Angle ayv is equal to yzx . Therefore the Angles yzv abx will be equal, Therefore the Lines yz ab will be parallel, which was to be done.

The following Problem is taken out of the second Book.

P R O B L E M.

The Line ad between b and c being Divided in b and c , to Divide it again in x so that the Rectangle axb be to the Rectangle $dx c$ as mp to gp .



A N A L Y S I S.

Let therefore	axb	$dx c ::$	$mp,$	gp
Therefore if you make	$ax,$	$xd ::$	$mp,$	py
And also	$bx,$	$xc ::$	py	gp

The Problem will be solv'd, for the products of the Analogous Terms will restitute the Proportion.

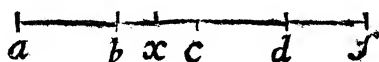
Let therefore	$ax,$	$xd ::$	$mp,$	py
and <i>Componendo</i>	$ax,$	$ad ::$	$mp,$	my
Let mg, mp, ad, ak be proportional	ak	mg		
Let also	$bx,$	$xc ::$	$py,$	gp
and <i>Componendo</i>	$bc,$	$xc ::$	$gy,$	gp
Let bc, cf, mg, gp be proportional	cf	mg		
Therefore <i>Componendo</i>	$xf,$	$xc ::$	$my,$	mg
and by equality	$xf,$	$xc ::$	$ak,$	ax
and <i>Convertendo</i>	$xf,$	$cf ::$	$ak,$	xk

The following Problem is taken out of the third Book.

The

P R O B L E M.

The Line ac being divided any where in b , to divide it again in x between b and c so that the Rectangle axb shall be equal to the Rectangle bxc together with the double square of xc .



A N A L Y S I S.

Let therefore	axb	$= bxc + 2xcx$
But by 3. 2. El.	bcx	$= bxc + xcx$
Therefore	axb	$= bcx + xcx$
Let cd be made $= bc$, therof.	bcx	$= dcx$
Therefore	axb	$= dcx = + xcx$
that is by 3. 2. El.	axb	$= dxc$
Therefore	ax, xc	$:: xd, bx$
and <i>Compendo</i>	ax, xc	$:: db, bx$
Let cf be made $= bd$		cf
and as the sum of the Antecedents, to the sum of the Consequents. So one Antecedent to its Consequent.		
Therefore	af, bc	$:: cf, bx$
Therefore the Problem is solv'd.		

Construction and Demonstration.

Let cd and df be made equal to bc , and let af, bc, cf, bx , be proportional, I say the thing is done.

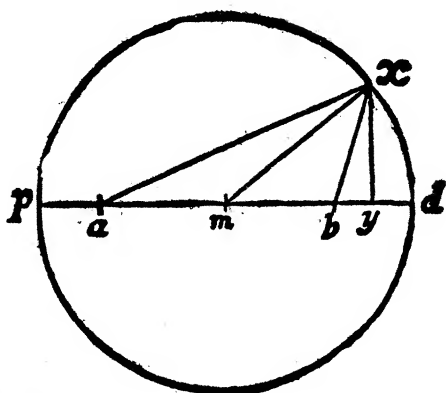
For since $af, bc :: cf, bx$, and the difference of the Antecedents to the difference of the Consequents as one Antecedent is to its Consequent, ac will be to xc , as cf or bc to bx , and the Rectangle axb will be equal to the Rectangle dxc , that is, to the Rectangle dcx together with the Square of xc or (because bc and cd are equal) to the Rectangle bcx with the Square of xc ; But the Rectangle bcx is equal to the Rectangle bxc and the Square of xc : Therefore the Rectangle axb is equal to the Rectangle bxc , and the double Square of xc . Which was to be done.

The following Proposition is taken out of the 4th. Book.

P R O B L E M.

Two Points a and b being given, to draw the two
Lines

Lines ax xb , whose Squares together shall be equal to the Square given gg .



Let axb whose height is xy be the Triangle required. Bisect ab in m and draw mx .

A N A L Y S I S.

Let therefore

$$axa + xbx = gg$$

But by the 13th. of the Introd. $axa + xbx = 2ama + 2mxm$

Therefore

$$gg = 2ama + 2mxm$$

or

$$gg - 2ama = 2mxm$$

Therefore the Problem is solv'd, but the Length of mx being given and not its Position, it is evident that it may be the Semidiameter of a Circle whose Circumference shall be the *Locus* of the point x .

Construction and Demonstration.

From the Square given gg Subtract the double Square of am , the Square root of half the remainder shall be the line mx , with the Center m and distance mx , describe the Circle pxd , I say that any point x taken in its Circumference resolves the Problem.

For since the double of the Squares of am and xm is equal to the Square gg , by the Construction, and by the 13th. Proposition of the Introduction to the Squares ax and xb : The two Squares ax and xb together will be equal to the Square gg . Which was to be done.

F I N I S.

E R R A T A.

P Age 355. l. r. for IV. r. III. p. 356. l. 26. for III. r. IV. and for *subtract*, *subtraction*, &c. r. *subtract*, &c. p. 357. l. 33. r. *Sofigenes*.